UNIT-I

PULSE DIGITAL MODULATION

Communication is the process of establishing connection or link between two points for information exchange.

Depending upon the message signal, communication is classified as under:

1. Analog Communication
2. Digital Communication.

Analog Communication

In Analog Communication the modulating signal is analog in nature. This analog message signal may be obtained from sources such as speech, video shooting, etc.

The analog message signal modulates some high carrier frequency inside the transmitter to produce modulated signal. This modulated signal is then transmitted with the help of a transmitting antenna to travel through the transmission channel.

At the receiver, this modulated signal is received and processed to recover the original message signal.

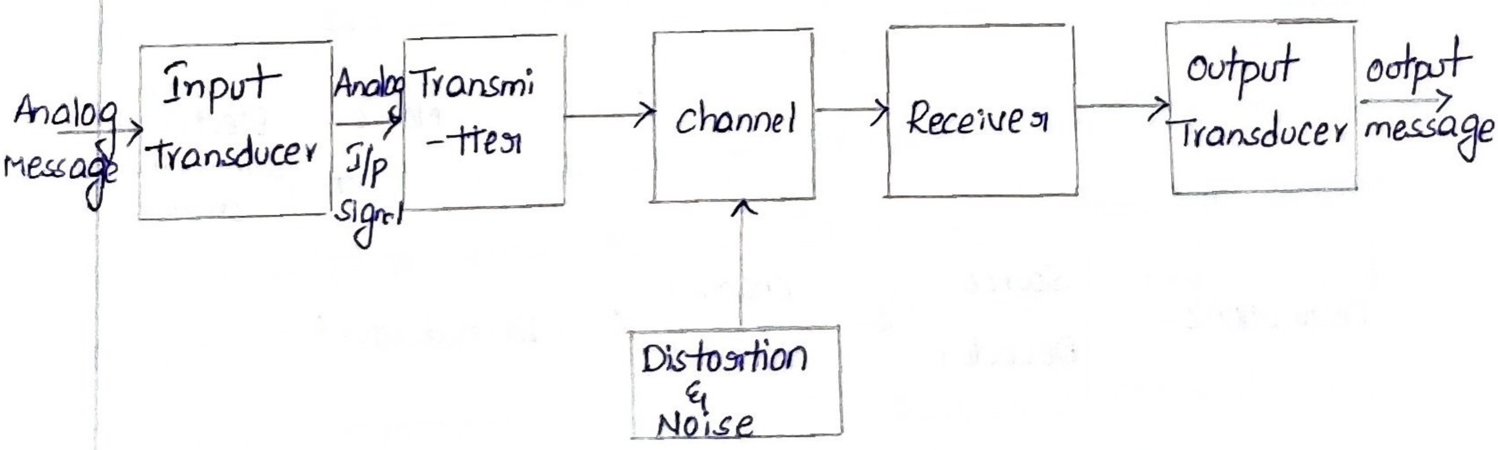


Fig1: Basic Analog Communication system

Digital Communication

In digital Communication, the message signal to be transmitted in Digital in nature.

Elements of a Digital Communication System

The below figure shows the model of a digital Communication System. The overall purpose of the system is to transmit the message or sequence of symbols coming out of a source to a destination point at as high rate and accuracy as possible.

The source and destination point are physically separated in space and a communication channel connects the source to the destination point.

The Communication channel accepts electrical (i.e electromagnetic) signals and the output of the channel is usually a distorted version of the input due to the non-ideal nature of the Communication channel.

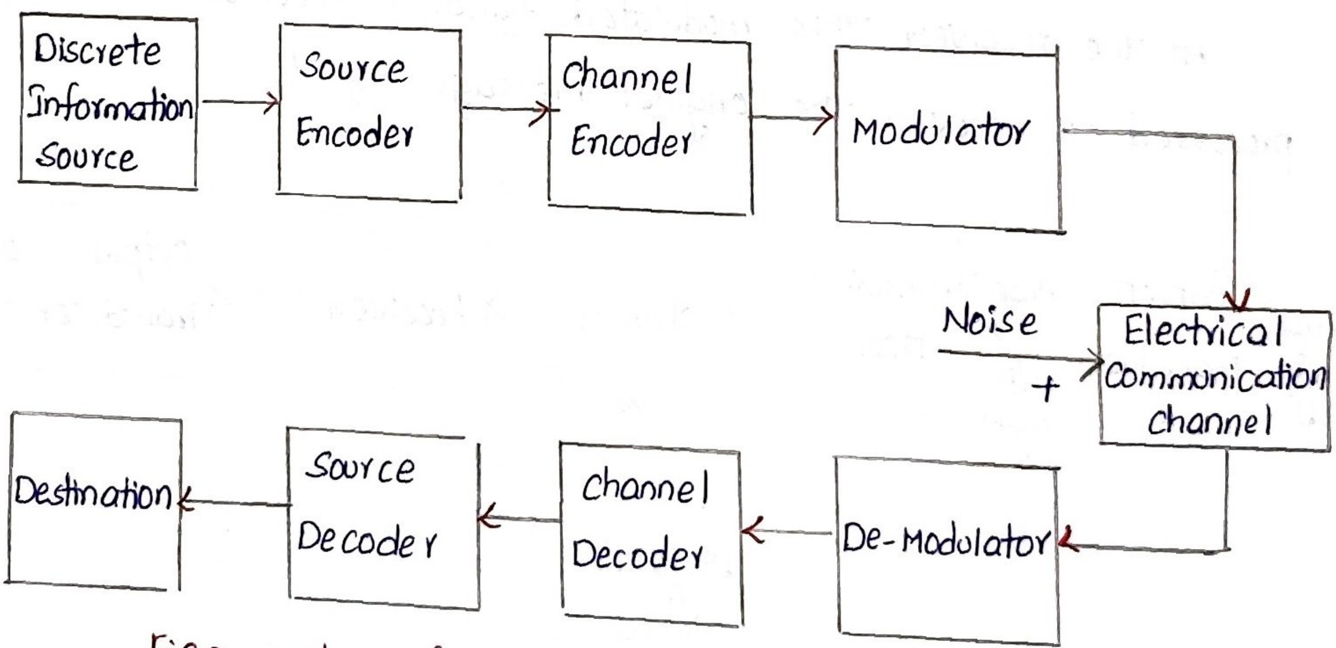


Fig 2: Model of a Digital Communication system

Discrete Information Source

An Analog information source may transformed into a discrete information source through the process of sampling and Quantizing. Discrete information sources are characterized by the following parameters:

1. **Source Alphabet:** These are the letters, digits or special characters available from the information source.

2. **Symbol rate :** It is the rate at which the information source generates source alphabet. It is generally represented in Symbols / sec unit.

3. **Source alphabet probabilities:** Each source alphabet from the source has independent occurrence rate in the sequence. As an example, letters A, E, I etc occur frequently in the sequence. Hence, probability of the occurrence of each source alphabet can become one of the important property which is useful in Digital Communication.

4. **Probablistic dependence of symbols in a sequence :** The information carrying capacity of each source alphabet is different in a particular sequence.

This parameter defines average information content of the symbols. The Entropy of a source describes the average information content per symbol in long messages. Entropy is measured in bits per symbol.

Information rate (I) is the product of symbol rate and source Entropy i.e

$$\underset{(\text{Bits/sec})}{\text{Information rate}} = \underset{(\text{symbol/sec})}{\text{Symbol rate}} \times \underset{(\text{Bits/symbol})}{\text{source entropy}}$$

Source Encoder

The symbols produced by the information source are given to the source encoder. They are first converted into digital form (i.e binary sequence of 1's & 0's) by the source encoder.

Each binary '1' and '0' is known as a bit. The group of bits is called a "code word". The source encoder assigns codewords to the symbols. For each distinct symbol, there is an unique code word.

The codeword can be of 4, 8, 16 or 32 bits length. As the number of bits are increased in each codeword, the symbols that may be represented are also increased.

Some typically source encoders are pulse code modulators, Delta modulators, vector quantizers etc. Source encoders have some following parameters:

1. **Block Size:** Block size describes the maximum no. of distinct codewords which can be represented by a source encoder. This depends on the number of bits in the code word. As an example, the block size of 8-bits source encoder will be 2^8 i.e 256 codewords.

2. **Codeword Length:**

Codeword length is the no. of bits used to represent each codeword. As an example, if 8-bits source encoder will be 2^8, i.e, 256, codewords are assigned to each codeword, then the codeword length will be 8-bits.

3. **Average Data rate:**

Average data rate is the output bits per second from the source encoder. The source encoder assigns multiple no. of bits to each input symbol. Hence data rate is generally higher than the symbol rate.

Data rate = Symbol rate x codeword length

= 10 X 8

= 80 bits/second

4. Efficiency of the Encoder:

It is the ratio of minimum source information rate to the actual output data rate of the source encoder.

Channel Encoder and Decoder

After converting the message or information signal in the form of binary sequence by the source encoder, the signal is transmitted through the channel. The communication channel adds noise and interference to the signal being transmitted. Hence errors are introduced in the binary sequence received at the receiver end.

Thus, channel coding is done to avoid these type of errors. Infact, the channel encoder adds some redundant binary bits to the input sequence.

A channel Encoder must have the following important parameters.

1. The coding rate that depends upon the redundant bits added by the channel encoder.

2. The coding Method used.
3. Coding efficiency which is the ratio of data rate at the input to the data rate at the output of the encoder.
4. Error control capabilities
5. Feasibility of the encoder and decoder.

Digital Modulators and De-Modulators

Digital continous wave modulators are ASK, FSK, PSK, DPSK. These modulators use a continous wave therefore they are also known as Digital continous Wave Modulators.

At the receiver end the digital demodulator converts the input modulated signal into the sequence of binary bits.

A Digital Modulation method must have following important parameters.

1. Bandwidth needed to transmit the signal
2. Probability of symbol or bit error
3. Synchronous & Asynchronous method of detection
4. Complexity of implementation.

Communication channel:

The connection between transmitter and receiver is established through a communication channel. The communication can take place through wireless, wirelines or fiber optic channel. Each and Every communication channel has some inherent problems. They are

1. Signal Attenuation: The signal attenuation in channel occurs due to the internal resistance of the channel and fading of the signal.

2. Amplitude and Phase distortion: The transmitted signal is distorted in amplitude and phase due to the non-linear characteristics of the communication channel.

3. Additive Noise interference: It is produced due to internal solid state devices and resistors etc.,

4. Multipath distortion: It occurs mostly in wireless communication channels.

→ Advantages and Disadvantages of Digital Communication

Advantages:

1. Digital Communications are simpler and cheaper compared to analog communications systems because of the advances made in the IC technologies.

2. In digital communications, the speech, video and other data may be merged and transmitted over a common channel using multiplexing

3. Using data Encryption, only permitted receivers may be allowed to detect the transmitted data. This property is mostly used in military applications.

4. Since the transmission is digital and the channel encoding is used, Therefore the noise does not accumulate from repeater to repeater in long distance communications.

5. The transmitted signal is digital in nature, therefore a large amount of noise interference may be tolerated.

6. In digital communication, channel coding is used, Therefore the errors may be detected and corrected in the receivers.

7. Digital Communications is adaptive to other advanced branches of data processing such as digital signal processing, image processing and data compression, etc.

Disadvantages:

1. Due to analog to digital conversion, the data rate becomes high. Therefore more transmission bandwidth is required for digital communication.

2. Digital communication needs synchronization in case of synchronous modulation.

→ **Comparison between Analog and Digital Modulation**

Analog Modulation	Digital Modulation
1. Transmitted modulated signal is analog in nature	1. Transmitted signal is digital i.e train of digital pulses.
2. Amplitude, frequency or phase variations in the transmitted signal represent the information or message.	2. Amplitude, width or position of the transmitted pulses is constant. The message is transmitted in the form of code words.
3. Noise immunity is poor for AM, but improved for FM and PM	3. Noise immunity is excellent.
4. It is not possible to separate out noise and signal. Therefore repeaters cannot be used.	4. It is possible to separate signal from noise. Therefore, repeaters can be used.
5. Coding is not possible	5. Coding techniques can be used to detect and correct the errors.

6. Bandwidth required is lower than that for the digital Modulation methods.	6. Due to higher bit rates, higher channel Bandwidth is required.
7. FDM is used for multiplexing	7. TDM is used for Multiplexing
8. Not suitable for transmission of secret information in Military applications.	8. Due to coding techniques, it is suitable for military applications.
9. Analog modulation systems are AM, FM, PM, PAM, PWM etc.	9. Digital Modulation systems are PCM, DM, ADM, DPCM etc.

→ PULSE CODE MODULATION

Pulse code Modulation is the name given to the class of baseband signals obtained from the quantized PAM signals by encoding each quantized sample into a Digital word. For baseband transmission, the codeword bits are transformed to pulse waveforms.

Definition:

Pulse code modulation is known as Digital pulse modulation technique. When pulse modulation is applied to a binary symbol, the resulting binary waveform is called Pulse - code modulation (PCM) waveform.

Elements of a PCM system:

The Fig1 shows the basic elements of a PCM system. It consists of three main parts i.e transmitter, transmission path and receiver.

The essential operations in the transmitter of a PCM system are sampling, quantizing and coding as shown in Fig1.

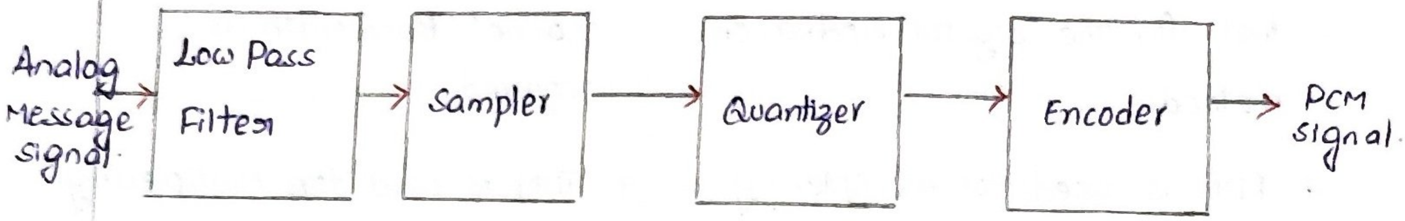


Fig 1(a): PCM transmitter

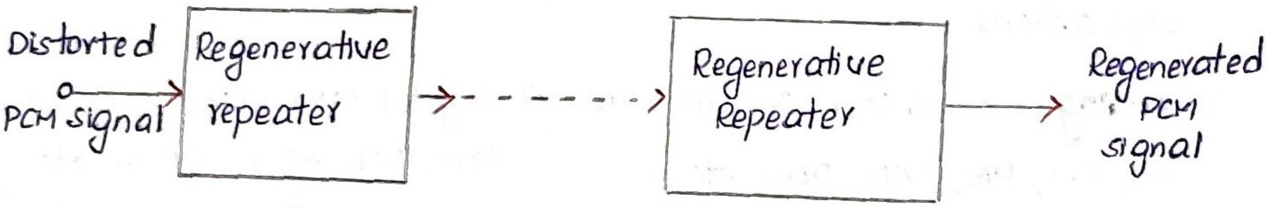


Fig 1(b): Transmission path

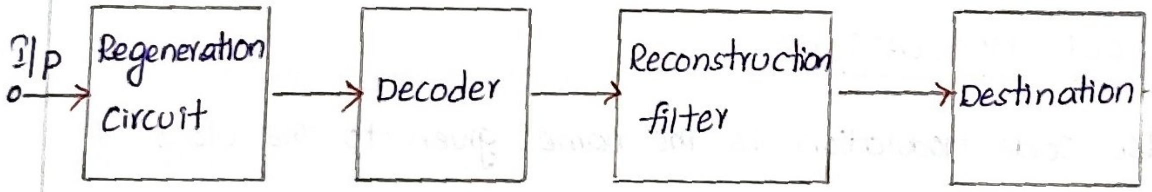


Fig 1(c): PCM Receiver

Fig 1: Elements of PCM system

Sampling is the operation in which an analog (i.e continous -time) signal is sampled ~~according~~ according to the sampling theorem resulting in a discrete time signal.

The quantizing and encoding operations are usually performed in the same circuit which is known as Analog-to-Digital converter (ADC).

The essential operations in the receiver are regeneration of impaired signals, decoding and demodulation of the train of quantized samples. These operations are performed in the same circuit known as Digital-to-analog converter (DAC).

In the transmission route from the transmitter to the receiver, regenerative repeaters are used to reconstruct the transmitted sequence of coded pulses in order to combat the accumolated effects of signal distortion and noise.

→ PCM GENERATOR OR TRANSMITTER

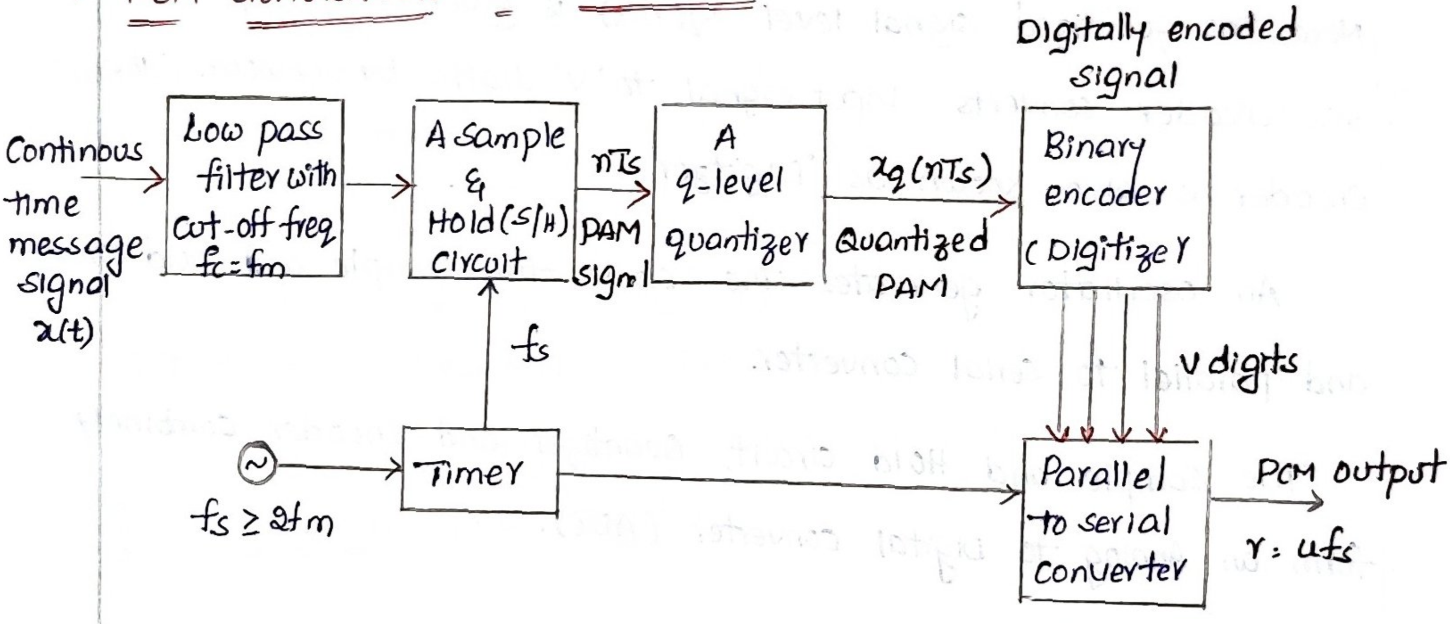


Fig: 3 : A practical PCM generator

In PCM generator the signal $x(t)$ is first passed through the low-pass filter of cut-off frequency f_m Hz. This low pass filter blocks all the frequency components which are lying above f_m Hz.

The sample and Hold circuit then samples this signal at the rate of f_s. sampling frequency f_s is selected sufficiently above nyquist rate to avoid aliasing i.e

$$f_s \geq 2f_m$$

The output of sample and hold circuit is denoted by $x(nT_s)$. This signal $x(nT_s)$ is discrete in time and continuous in amplitude.

A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It results minimum distortion or Error. This error is called "Quantization Error".

Thus, output of quantizer is a digital level called $x_q(nT_s)$. Now, the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word. This encoder is also known as "Digitzer".

An oscillator generates the clocks for sample and hold circuit and parallel to serial converter.

The sample and Hold circuit, Quantizer and encoder combinely form an Analog to Digital converter (ADC).

→ PCM TRANSMISSION PATH

The path between the PCM transmitter and PCM receiver over which the PCM signal travel is called as PCM transmission path and it is as shown in fig 4.

The most important feature of PCM system lies in its ability to control the effects of distortion and noise when the PCM wave travels on the channel.

PCM accomplishes this capacity by means of using a chain of regenerative repeaters as shown in fig4. Such repeaters are spaced close enough to each other on the transmission path.

The Regenerative performs three basic operations namely, equalization, timing and decision-making. Hence each repeater actually reproduces the clean noise free PCM signal from the PCM signal distorted by the channel noise. This improves the performance of PCM in presence of noise.

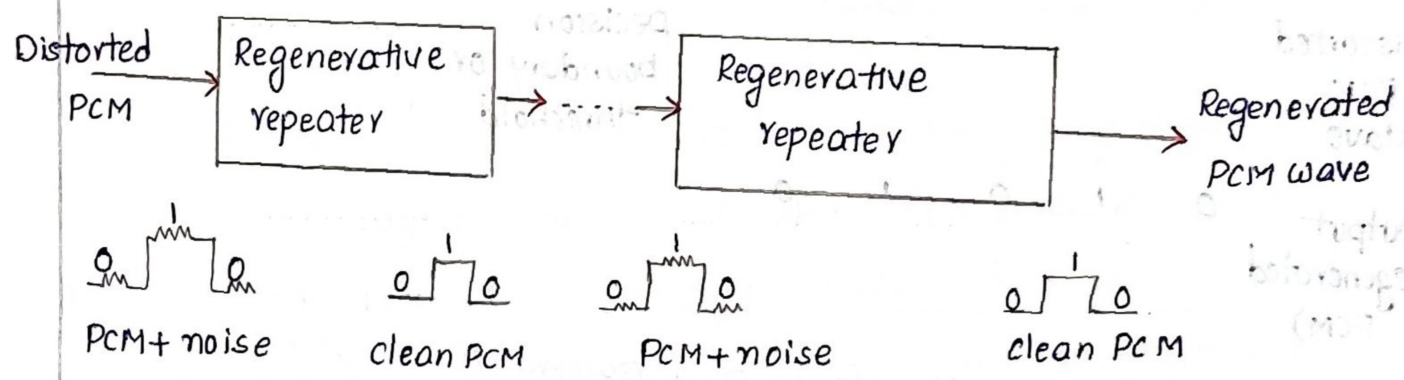


Fig 4(a): PCM transmission path

Block diagram of a Repeater

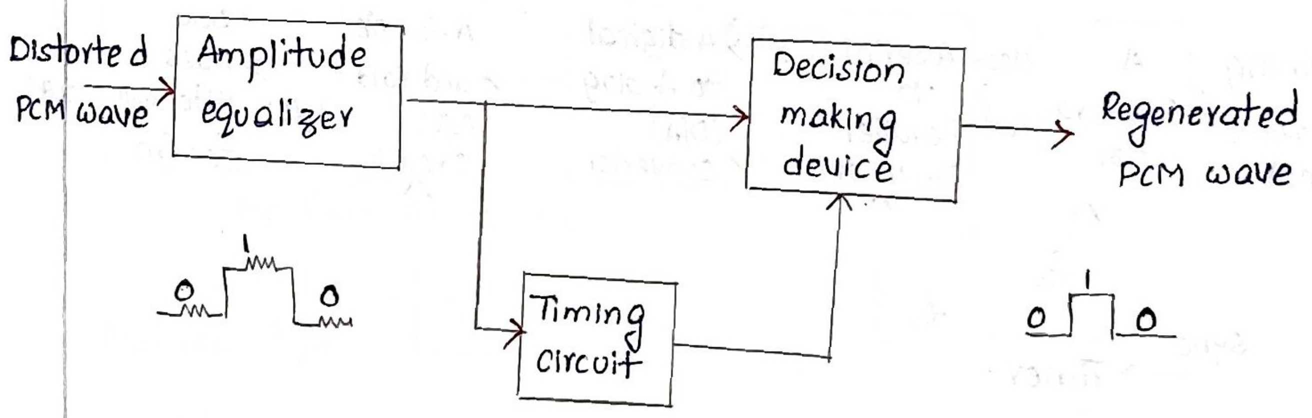


Fig 4(b): Block diagram of a Regenerative repeater

The amplitude equalizer shapes the distorted PCM wave so as to compensate for the effects of amplitude and phase distortions. The timing circuit produces a periodic pulse train which is derived from the input PCM pulses. The pulse train is then applied to decision-making device. The decision device makes a decision about whether the equalized PCM wave at its input has a '0' value or '1' value at the instant of sampling. Such a decision is made by comparing equalized PCM with a reference level called decision threshold as shown in Fig 4(c). At the output of the decision device, we get a clean PCM signal without any trace of noise.

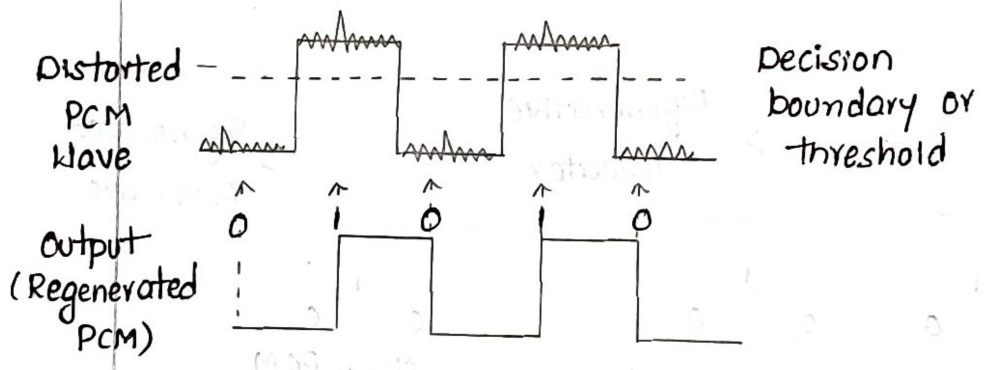


Fig 4(b): Waveforms of a regenerative repeater.

→ PCM RECEIVER

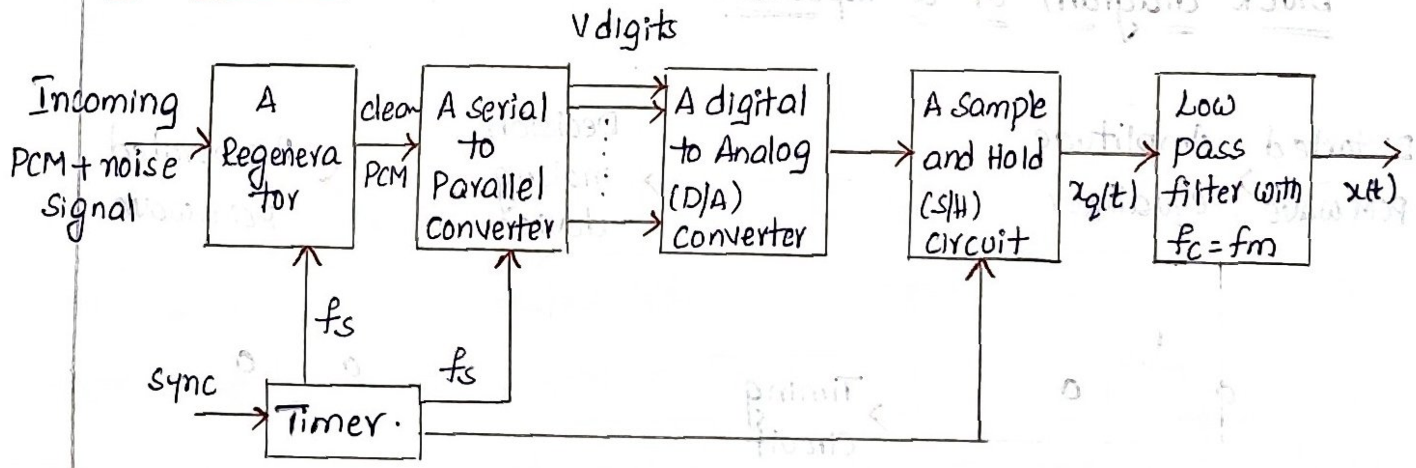


Fig 5(a): PCM Receiver

The above fig shows the block diagram of PCM receiver. The regenerator at the start of PCM receiver reshapes the pulse and removes the noise. This signal is then converted to parallel digital words for each sample.

Now, the digital word is converted to its analog value denoted as $x_q(t)$ with the help of sample and hold circuit.

This signal, at the output of sample and hold circuit, is allowed to pass through a low pass reconstruction filter to get the appropiate original message signal denoted as $x(t)$

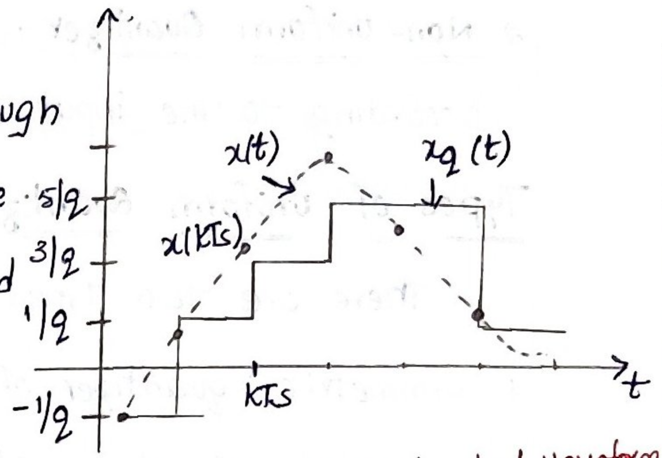


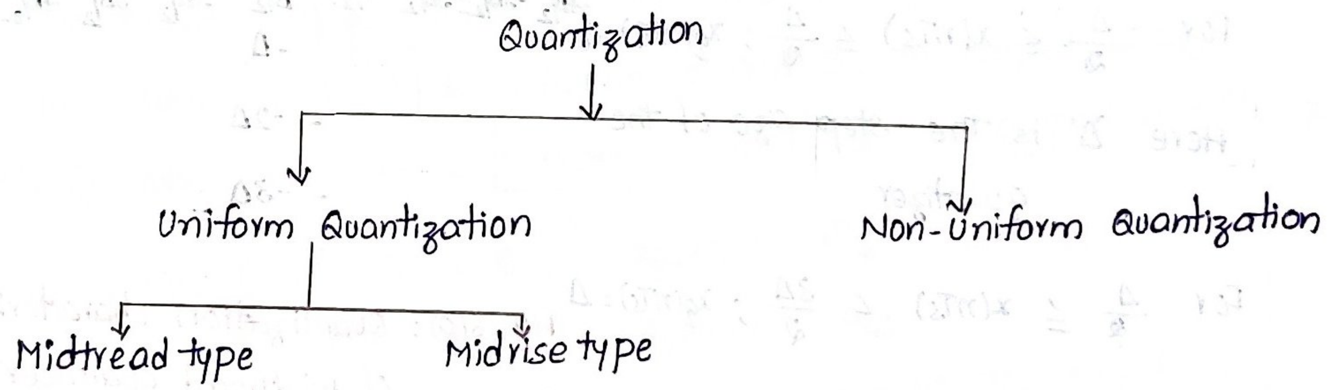
Fig 5(b): Reconstructed Waveform

→ QUANTIZER

A q-level quantizer compares the discrete-time input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels, which results in minimum distortion or error. This error is called "Quantization Error".

Classification of Quantization Process

Basically, quantization process may be classified as follows:



The quantization process can be classified into two types as under:

1. Uniform Quantization
2. Non-uniform Quantization.

This classification is based on the step size

1. Uniform Quantizer: In this Quantizer the step size remains same throughout the input range.

2. <u>Non- uniform Quantizer</u> : In this quantizer the step size varies according to the input signal values.

Types of uniform Quantizer

There are two types of uniform quantizer as under :

1. Symmetric quantizer of the Midtread type
2. Symmetric quantizer of the Midrise type.

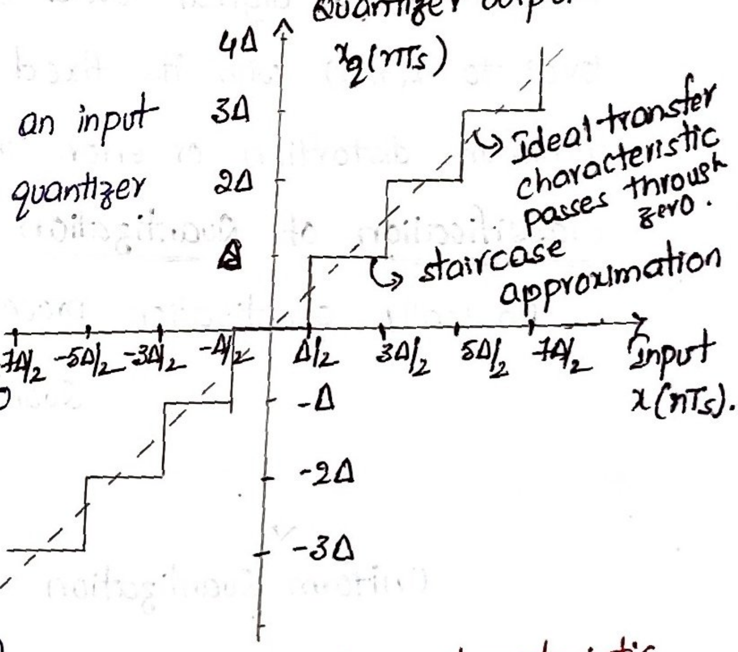
Midtread Quantizer :

The transfer characteristic of the midtread quantizer is shown in Fig 6(a).

As shown in this figure, when an input is between $-\Delta/2$ to $+\Delta/2$ then the quantizer output is zero i.e

For $\frac{-\Delta}{2} \leq x(nT_s) \leq \frac{\Delta}{2}$; $x_q(nT_s) = 0$

Here 'Δ' is the step size of the quantizer



For $\frac{\Delta}{2} \leq x(nT_s) < \frac{3\Delta}{2}$; $x_q(nT_s) = \Delta$

Fig 6(a): Quantization characteristic of midtread quantizer

Similarily other levels are assigned.

It is called midtread because quantizer output is zero when $x(nT_s)$ is zero. Fig 6(b) shows the quantization error of midtread quantizer. Quantization error is given as,

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

In Fig 6(b) observe that when $x(nT_s) = 0$, $x_q(nT_s) = 0$. Hence quantization is zero at origin. When $x(nT_s) = \Delta/2$, quantizer output is zero just before this level. Hence error is $\Delta/2$ near this level. From fig 6(b) it is clear that

$$-\Delta/2 \leq \varepsilon \leq \Delta/2$$

Thus quantization error lies between $-\Delta/2$ and $\Delta/2$. And maximum quantization error is $\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$

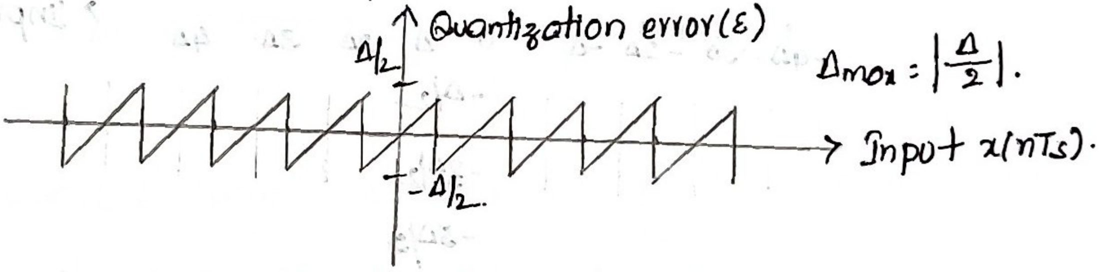


Fig 6(b): Quantization Error

Midriser Quantizer:

The transfer characteristic of the midriser quantizer is shown in Fig 7(a). When an input is between 0 and Δ, the output is $\Delta/2$. Similarily when an input is between 0 and $-\Delta$, the output is $-\Delta/2$ ie

$$\text{For } 0 \leq x(nT_s) < \Delta; \quad x_q(nT_s) = \Delta/2$$
$$-\Delta \leq x(nT_s) < 0; \quad x_q(nT_s) = -\Delta/2$$

Similarily when an input is between 3Δ and 4Δ, the output is $7\Delta/2$. This is called midriser quantizer because its output is either $-\Delta/2$ or $-\Delta/2$ when input is zero.

Fig 7(b) shows the quantization error in midriser quantization. Hence quantization error will be

$$\varepsilon = x_q(nT_s) - x(nT_s)$$
$$= \frac{\Delta}{2} - 0 \quad = \frac{\Delta}{2}$$

Thus, the quantization error lies between $-\Delta/2$ & $\Delta/2$ ie

$$-\Delta/2 \leq \varepsilon < \Delta/2$$

And the maximum quantization error is

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right|.$$

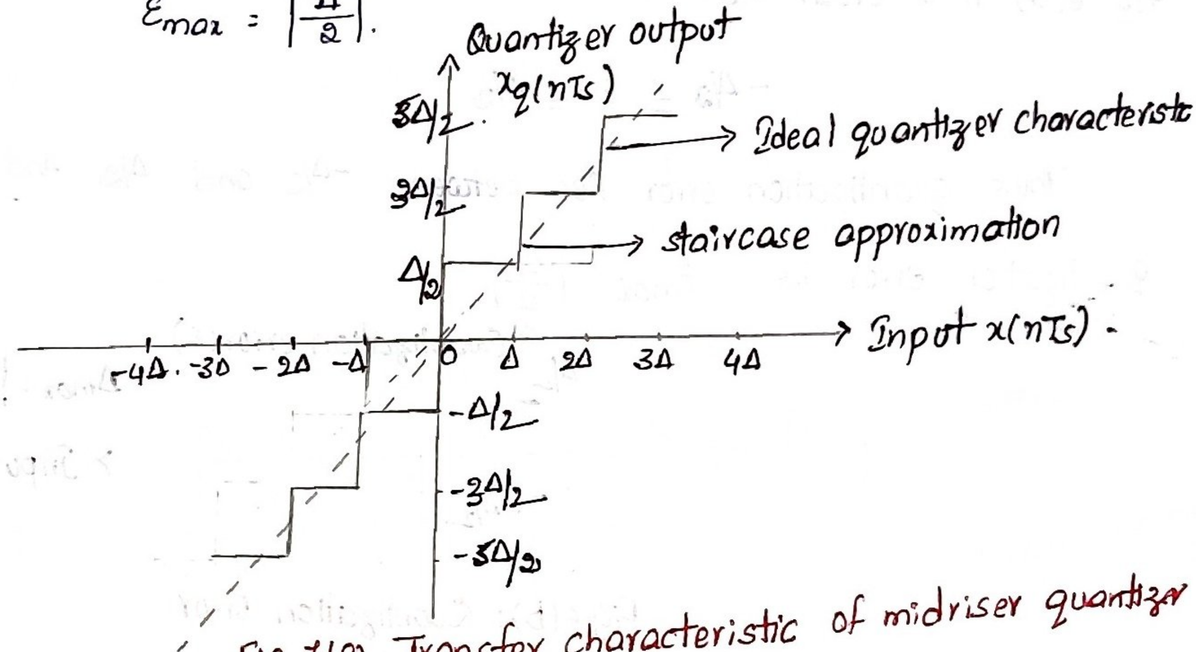


Fig 7(a). Transfer characteristic of midriser quantizer

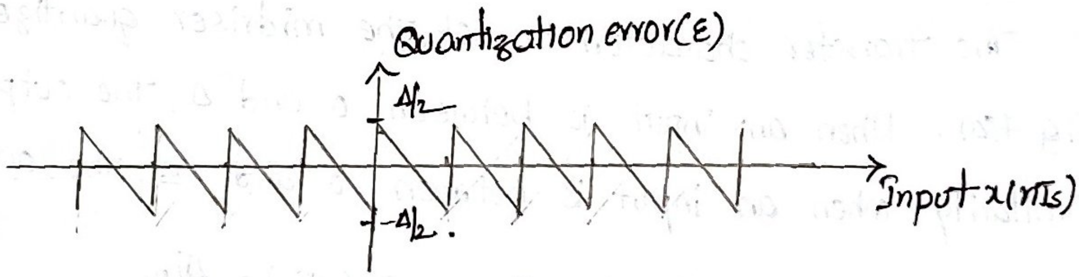


Fig 7(b): Quantization Error.

→ Quantization Noise and Signal to Noise Ratio in PCM

1. **Quantization Error:** Because of quantization, inherent errors are introduced in the signal. This error is called quantization error.

The quantization error can be defined as

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

2. **Step size**: Let an input $x(nT_s)$ be of continous amplitude in the range $-x_{max}$ to x_{max}. From Fig 7(a) we know that the total excursion of input $x(nT_s)$ is mapped into 'q' levels on vertical axis. That is when input is 3Δ, output is $5\Delta/2$ and when the input is -3Δ, the output is $-5\Delta/2$. That is $+x_{max}$ represents $5\Delta/2$ and $-x_{max}$ represents $-5\Delta/2$.

Therefore the total amplitude range becomes,

$$\text{Total amplitude range} = x_{max} - (-x_{max})$$
$$= 2x_{max}$$

If this amplitude range is divided into 'q' levels of quantizer then the step size is given as

$$\Delta = \frac{2x_{max}}{q}$$

If the signal is normalized to minimum and maximum values equal to 1, then $x_{max} = 1$; $-x_{max} = -1$

Therefore step size will be

$$\Delta = \frac{1-(-1)}{q} = \frac{2}{q} \text{ (for normalized signal)}$$

3. **Probability density function (Pdf) of Quantization error**

If step size 'Δ' is sufficiently small, then it is reasonable to assume that the quantization error 'Δ' will be uniformly distributed random variable. The maximum quantization error is given by

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right| \quad \text{i.e}$$

$$-\frac{\Delta}{2} < \varepsilon_{max} < \frac{\Delta}{2}$$

Thus over the interval $\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$ quantization error is uniformly distributed random variable

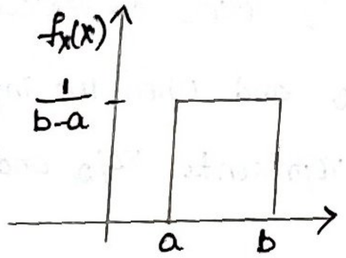


Fig 8(a): uniform distribution

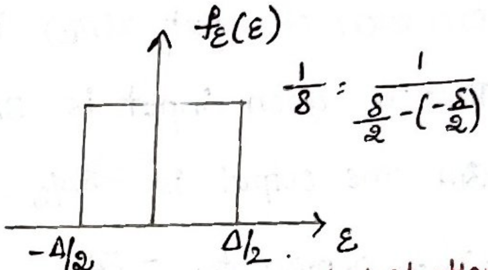


Fig 8(b): Uniform distribution for quantization error.

In above figure, a random variable is said to be uniformly distributed over an interval (a,b). Then PDF of 'x' is given by

$$f_X(x) = \begin{cases} 0 & \text{for } x \le a \\ \frac{1}{b-a} & \text{for } a < x \le b \\ 0 & \text{for } x > b \end{cases}$$

Thus, with the help of above equation we can define the probability density function for quantization error 'ε' as

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon < -\Delta/2 \\ \frac{1}{\Delta} & \text{for } -\Delta/2 < \varepsilon < \Delta/2 \\ 0 & \text{for } \varepsilon > \Delta/2 \end{cases}$$

4. **Noise Power**

From fig 8(b). we can see the quantization error 'ε' has zero average value. That is mean 'm_ε' of the quantization error is "zero".

The signal to Quantization noise ratio of the quantizer is defined as

$$\frac{S}{N} = \frac{\text{signal power (normalized)}}{\text{Noise power (normalized)}}$$

If type of signal at input i.e $x(t)$ is known, then it is possible to calculate signal power.

The noise power is given as

$$\text{Noise power} = \frac{V^2_{noise}}{R}$$

Here V^2_{noise} is the mean square value of noise voltage. Since noise is defined as random variable 'ε' and PDF $f_\varepsilon(\varepsilon)$ its mean square value is given as

$$\text{mean square value} = E[\varepsilon^2] = \bar{\varepsilon}^2$$

The mean square value of a random variable 'X' is given as

$$\bar{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

$$= \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \cdot \frac{1}{\Delta}\, d\varepsilon = \frac{1}{\Delta}\left[\frac{\varepsilon^3}{3}\right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right] = \frac{1}{3\Delta} \cdot \frac{2\Delta^3}{8} = \frac{\Delta^2}{12}$$

$\therefore$ The mean square value of noise voltage is $\frac{\Delta^2}{12}$.

When load Resistance, $R = 1\Omega$, then the noise power is normalized i.e

$$\text{Noise power (normalized)} = \frac{V^2_{noise}}{R} = \frac{V^2_{noise}}{1}$$

$$= \frac{\Delta^2}{12}$$

→ Derivation of Maximum Signal to Quantization Noise Ratio for Linear Quantization:

The signal to quantization noise ratio is defined as

$$\frac{S}{N} = \frac{\text{Normalized Signal Power}}{\text{Normalized Noise power}}$$

$$= \frac{\text{Normalized signal Power}}{\Delta^2/12} \quad - (1)$$

The number of bits 'v' and quantization levels 'q' are related as

$$q = 2^v \quad - (2)$$

putting this value in $\Delta = \frac{2x_{max}}{q}$ - (3)

$$\Delta = \frac{2x_{max}}{2^v} \quad - (4)$$

substitute the equation (4) in eq (1), then we get

$$\frac{S}{N} = \frac{\text{Normalized signal Power}}{\left(\frac{2x_{max}}{2^v}\right)^2 \times \frac{1}{12}}$$

$$= \frac{\text{Normalized signal Power}}{\frac{4x_{max}^2}{2^{2v}} \times \frac{1}{12}}$$

Let normalized signal power be denoted as 'P'

$$\frac{S}{N} = \frac{3P}{x_{max}^2} \cdot 2^{2v} \quad - (5)$$

The above equation shows that signal to Noise power ratio of quantizer increases exponentially with increasing bits per sample

If we assume that input $x(t)$ is normalized i.e

$$x_{max} = 1$$

Then signal to quantization noise ratio will be

$$\frac{S}{N} = 3P \cdot 2^{2v}$$

If the destination signal power 'P' is normalized i.e

$$P \leq 1$$

Then the signal to noise ratio is given as

$$\frac{S}{N} \leq 3 \times 2^{2v}$$

Expressing the signal to Noise ratio in decibels,

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\leq 10\log_{10}\left(3 \times 2^{2v}\right)$$

$$\leq 10\log_{10} 3 + 10\log_{10} 2^{2v}$$

$$\leq 4.8 + 20v\,(0.3)$$

$$\boxed{\left(\frac{S}{N}\right)_{dB} \leq (4.8 + 6v)\,dB.}$$

→ **Signal to Quantization Noise ratio for PCM system**

Let us assume that the modulating signal be a sinusoidal voltage, having peak amplitude A_m.

The power of this signal will be $P = \frac{V_{rms}^2}{R}$

$$= \left(\frac{A_m}{\sqrt{2}}\right)^2 \quad (\because R = 1\Omega)$$

Normalized power $= \frac{A_m^2}{2}$ — (1)

We know that the signal to Quantization noise ratio is

$$\frac{S}{N} = \frac{3P}{x^2_{max}} \times 2^{2v} \quad —(2).$$

Substitute the eq (1) in (2) then

$$\frac{S}{N} = \frac{\frac{3 \times A_m^2}{2}}{A_m^2} \times 2^{2v}$$

$$= \frac{3}{2} \times 2^{2v}$$

$$= 1.5 \times 2^{2v}.$$

Expressing signal to Noise ratio in dB, take logarithm

$$\left(\frac{S}{N}\right)_{dB} = 10\log\left(\frac{S}{N}\right)$$

$$= 10\log(1.5 \times 2^{2v})$$

$$= 10\log(1.5) + 10\log(2^{2v})$$

$$= 10 \times 0.176 + 20v\log 2$$

$$= 1.76 + 20v(0.3)$$

$$= 1.76 + 6v$$

Thus,

$$\boxed{\left(\frac{S}{N}\right)_{dB} \text{ in PCM} = 1.8 + 6v; \text{ for sinusoidal signals.}}$$

→ TRANSMISSION BANDWIDTH IN A PCM SYSTEM.

Let us assume that the quantizer use 'v' no. of binary digits to represent each level. Then the no. of levels that may be represented by 'v' digits will be

$$q = 2^{v}$$

where 'q' represents the total no. of digital levels of a q-level quantizer.

Each sample is converted to 'v' binary bits i.e

Number of bits per sample $= v$

We know that the no. of samples per second $= f_s$

Therefore, No. of bits per second is expressed as

No. of bits per sec = No. of bits per samples × No. of samples per second

= v bits per sample × f_s samples per second —①

The no. of bits per second is known as signaling rate of PCM and is denoted by 'r' i.e

Signalling rate in PCM, $r = v f_s$ — ②

$$f_s = 2 f_m \quad \text{—③}$$

Substitute the eq ③ in eq ②, then

$$r = v \times 2 f_m$$

Since Bandwidth needed for PCM transmission is given by half of the signalling rate.

Transmission BW in PCM,

$$BW \geq \frac{1}{2} r$$

$$\geq \frac{1}{2} \times v \times 2 f_m$$

$$BW \geq v f_m$$

P.1 A Television signal having a bandwidth of 4.2MHz is transmitted using binary PCM system. Given that the no.of quantization levels is 512. Determine

(i) codeword length

(ii) Transmission Band Width

(iii) Final bit rate

(iv) output signal to quantization noise ratio

Sol. Given that bandwidth is 4.2 MHz. This means highest frequency component will have frequency of 4.2 MHz i.e

$$f_m = 4.2 MHz.$$

And quantization levels $= q = 512$

(i) We know that the number of bits and quantization levels are related in binary PCM as

$$q = 2^{v}$$

$$512 = 2^{v}$$

$$v = \log_2 512 = \log_2 2^9$$

$$v = 9 \text{ bits}$$

Hence the codeword length is 9 bits.

(ii) We know that the transmission channel bandwidth is given as,

$$BW \geq v f_m$$

$$\geq 9 \times 4.2 \times 10^6$$

$$\geq 37.8 MHz.$$

(iii) The final bit rate is equal to signalling rate

We know that the signalling rate is given as

$$r = v f_s$$

$$= 9 \times (2 f_m) \quad (\because f_s \geq 2 f_m)$$

$$= 9 \times 2 \times 4.2 \times 10^6$$

$$= 75.6 \times 10^6 \text{ bits/sec.}$$

(or)

$$BW \geq \frac{1}{2} r$$

$$r = 2 BW$$

$$= 2 \times 37.8 \times 10^6$$

$$= 75.6 \times 10^6 \text{ bits/sec.}$$

(iv) The output signal to noise ratio is expressed as

$$\left(\frac{S}{N}\right) dB \leq (4.8 + 6v) dB$$

But $v = 9$ bits

$$\left(\frac{S}{N}\right) \leq 4.8 + 6 \times 9$$

$$\leq 58.8 \text{ dB}.$$

P2 The bandwidth of an input signal to the PCM is restricted to 4 KHz. The input signal varies in amplitude from -3.8V to 3.8V and has the average power of 30mW. The required signal to noise ratio is given as 20dB. The PCM modulator produces binary output. Assuming uniform quantization.

(i) Find the no. of bits required per sample

(ii) Outputs of 30 such PCM coders are time multiplexed. What would be the minimum required transmission bandwidth for this multiplexed signal?

Sol. Given the signal to noise ratio is 20dB

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) = 20.$$

$$\frac{S}{N} = 10^2 = 100$$

(i) We know that the signal to quantization noise ratio is given as

$$\frac{S}{N} = \frac{3P \cdot 2^{2v}}{x_{max}^2}$$

$$100 = \frac{3 \times 30 \times 10^{-3} \times 2^{2v}}{(3.8)^2}$$

$$v = 6.98 \text{ bits}$$

$$= 7 \text{ bits}$$

(ii) The maximum frequency is given as

$$f_m = 4KHz.$$

We know that the transmission bandwidth is expressed as

$$BW \geq \nu f_m$$

Since there are 30 PCM coders which are time multiplexed, the transmission bandwidth must be.

$$BW \geq 30 \times \nu \times f_m$$

$$\geq 30 \times 7 \times 4$$

$$\geq 840KHz.$$

We know that signalling rate is twice the transmission Bandwidth ie.

$$r = 840 \times 2$$

$$= 1680 \text{ bits/sec}.$$

→ NECESSITY OF NON-UNIFORM QUANTIZATION IN A PCM SYSTEM

In uniform quantization, the quantizer has a linear characteristics. The step size remains same throughout the range of quantizer. Therefore, over the complete range of inputs, the maximum quantization error also remains same.

The maximum quantization error = $\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$

Also, the step size is expressed as

$$\Delta = \frac{2x_{max}}{q}$$

If $x(t)$ is normalized, its maximum value i.e $x_{max} = 1$

Therefore, we have $\Delta = \frac{2}{q}$

Let us consider an example of PCM system in which we take $\nu = 4$ bits

Then no. of levels q will be $q = 2^4 = 16$ levels.

Then step size Δ will be

$$\Delta = \frac{2}{q} = \frac{2}{16} = \frac{1}{8}$$

Hence quantization error $\varepsilon_{max} = \left|\frac{\Delta}{2}\right| = \left|\frac{1}{8\times 2}\right| = \left|\frac{1}{16}\right|$

Therefore the quantization error is $\frac{1}{16}$th part of the full voltage range.

We assume that full voltage range is 16 volts. Then maximum quantization error will be 1 volt.

For low signal amplitudes like 2V, 3V etc, the maximum quantization error of 1volt is quite high i.e about 30 to 50 %. This means that for signal amplitudes which are close to 15V or 16V etc, the maximum quantization error of 1volt can be considered to be small.

This problem arises because of uniform quantization. Therefore non-uniform quantization should be used in such cases.

→ NON-UNIFORM QUANTIZATION

If the quantizer characteristics is non-linear and the step size is not constant instead if it is variable, dependent on the amplitude of input signal then the quantization is known as "Non-uniform quantization."

For weak signals ($P \ll 1$), the step size is small, therefore the quantization noise reduces, to improve the signal to quantization noise ratio for weak signals.

The step size is thus Varied according to the signal level to keep the signal to noise ratio adequantely high. This is Non-Uniform Quantization.

The Non-Uniform quantization is practically achieved through a process called "Companding".

→ COMPANDING

Companding is a term derived from two words i.e compression and expansion as under:

Companding = Compressing + Expanding

In practice, it is difficult to implement the non-uniform quantization because it is not known in advance about the changes in the signal level. Therefore, a particular method is used.

The weak signals are amplified and strong signals are attenuated before applying them to a uniform quantizer. This process is called as compression and the block that provides it is called as "Compressor".

At the receiver exactly opposite is followed which is called expansion. The circuit used for providing expansion is called as an "expander".

The process of companding is shown in the form of block diagram as shown in figure 9. The compression of signal at the transmitter and expansion at the receiver is combined to be called "companding".

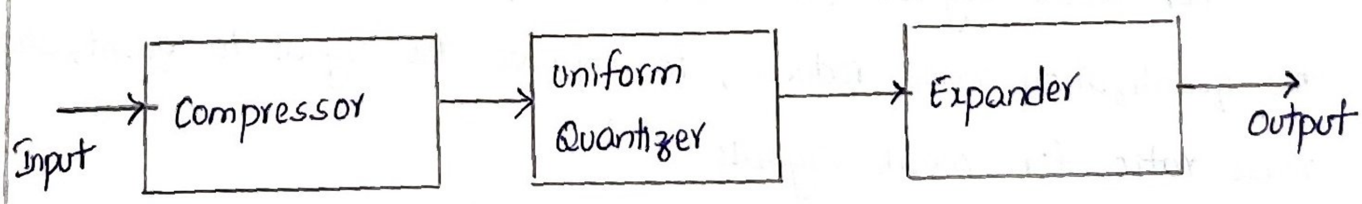


Fig: 9 Companding Model

Compressor characteristic

The standard telephone technique of handling the large range of possible input signal levels is to use a logarithmic-compressed quantizer instead of a uniform one.

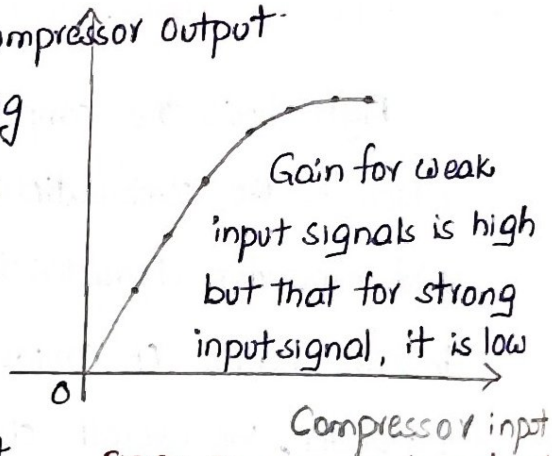


Fig10: Compressor characteristic

With such Non-uniform compressor, the output SNR is independent of the distribution of input signal levels. Fig 10 shows the compressor characteristics. As shown in Fig:10 the compressor provides a higher gain to the weak signals and smaller gain to the strong input signals. Thus, weak signals are artifically boosted to improve the signal to quantization noise ratio.

For small magnitude signals, the compression characteristic has a much steeper slope than for large magnitude signals. Thus, a given signal change at small magnitudes will carry the uniform quantizer through more steps than the same change at large magnitudes.

Expander characteristics

Fig 11 shows the expander characteristics. This characteristics is exactly the inverse of the compressor characteristics.

This ensures that all the artifically boosted signals by the compressor are brought back to their original amplitudes at the receiver end.

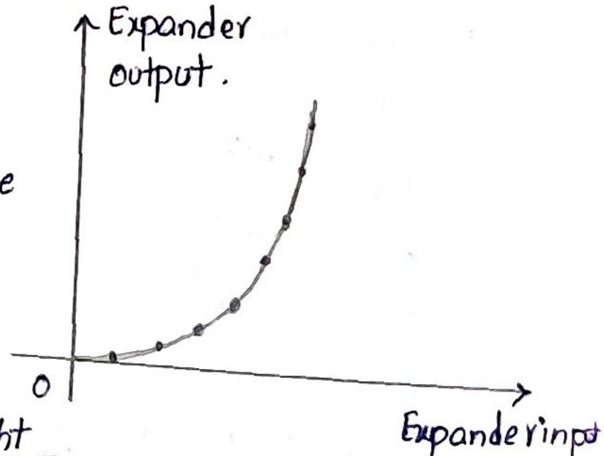


Fig 11: Expander characteristics

→ Compander characteristic

Fig12 shows the compander characteristics which is the combination of the compressor and expander characteristics. Due to the inverse nature of compressor and expander, the overall characteristics of the compander is a straight line (i.e dotted line in fig 12). This indicates that all boosted signals are brought back to their original amplitudes.

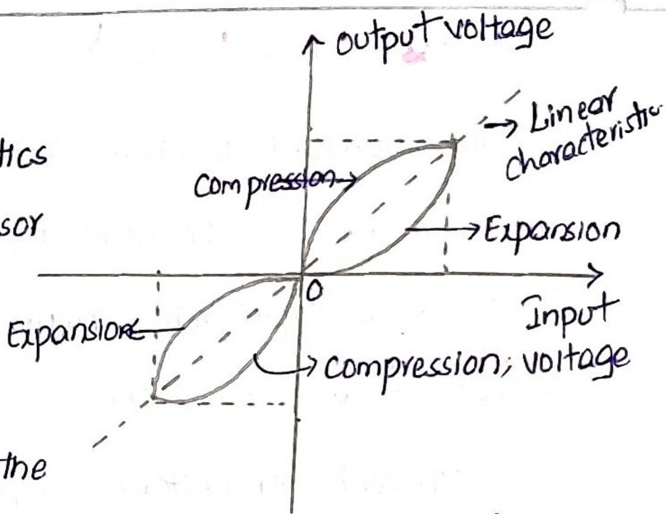


Fig12: Companding curves for PCM System

→ Different types of Compressor characteristics

We need a linear compressor characteristics for small amplitudes of the input signal and a logarithmic characteristics elsewhere. In practice, this is achieved by using following two methods:

1. μ-law companding
2. A-Law companding

μ-Law Companding:

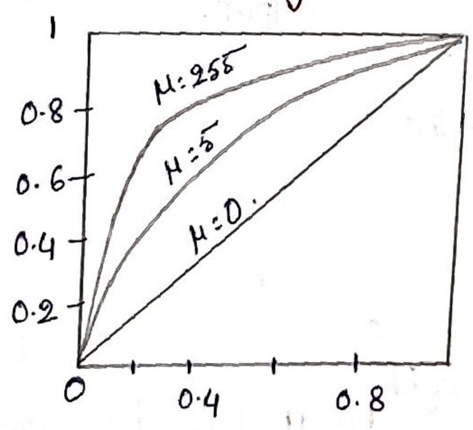
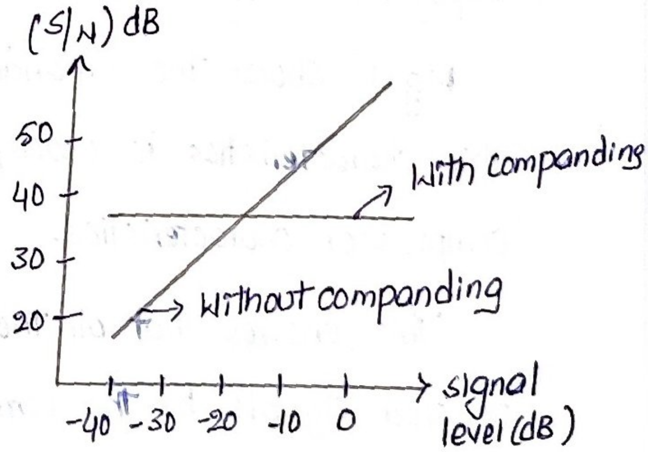


Fig 13 (a) compressor characteristics of a μ-law compressor



13 (b) PCM performance with μ-law Companding

μ-Law Companding

In the μ-Law Companding, the compressor characteristic is continous. It is approximately linear for smaller values of input levels and logarithmic for high input levels. The μ-law compressor characteristic is mathematically expressed as under:

$$z(x) = (\operatorname{sgn} x) \frac{\ln\left(1+\mu \frac{|x|}{x_{max}}\right)}{\ln(1+\mu)} \qquad \text{where } 0 \le |x|/x_{max} \le 1.$$

Here $z(x)$ represents the output and x is the input to the compressor.

$|x|/x_{max}$ represents the normalized value of input with respect to the maximum value x_{max}.

$(\operatorname{sgn})x$ term represents ± 1 i.e positive and negative values of input and output.

The μ-law compressor characteristics for different values of μ as shown in fig 13(a). The practically used value of μ is 255. It may be noted that the characteristic corresponding to $\mu = 0$ corresponds to the uniform quantization. The μ-law companding is used for speech and music signals. It is used for PCM telephone systems in united states, canada and Japan.

Fig 13(b) shows the variations of signal to quantization noise ratio with respect to signal level, with and without companding. It is obvious that SNR is almost constant at all the signal levels when companding is used.

A-Law Companding

In the A-Law companding, the compressor characteristic is piecewise, made up of a linear segment for low level inputs and a logarithmic segment for high level inputs. Fig 14 shows the A-law compressor characteristics for different values of A.

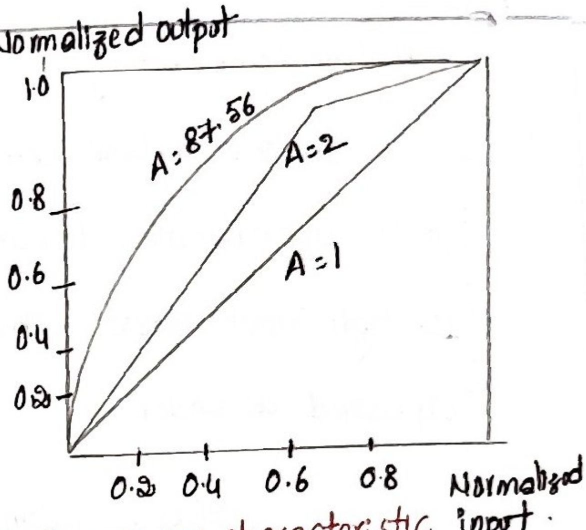


Fig 14: Compressor characteristic of A-law compressor

Corresponding to A=1, we observe that the characteristic is linear which corresponds to a uniform quantization. The practically used value of A is 87.56.

The A-Law companding is used for PCM telephone systems in Europe. It is mathematically expressed as under:

$$\frac{z(x)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\log_e A} & \text{for } 0 \leq \dfrac{|x|}{x_{max}} \leq 1 \\[2ex] \dfrac{1+\log_e\left[A|x|/x_{max}\right]}{1+\log_e A} & \text{for } \dfrac{1}{A} \leq \dfrac{|x|}{x_{max}} \leq 1. \end{cases}$$

Applications of PCM

1. With the advent of fibre optic cables, PCM is used in telephony.
2. In space communication, space craft transmits signals to earth. Here, the transmitted power is quite small (i.e 10 or 15W) and the distances are very large (i.e a few million km). However, due to the high noise immunity, only PCM systems can be used in such applications.

→ **Advantages of PCM :**

1. PCM provides high noise immunity
2. Due to digital nature of the signal, we can place repeaters between the transmitter and the receivers. Repeaters reduce the effect of noise.
3. We can store the PCM signal due to its digital nature
4. We can use various coding techniques so that only the desired person can decode the received signal.

→ **Drawbacks of PCM**

1. The encoding, decoding and quantizing circuitry of PCM is complex
2. PCM requires a large bandwidth as compared to the other systems.

→ **Delta Modulation**

In PCM, it transmits all the bits which are used to code a sample. Hence, signaling rate and transmission channel bandwidth are quite large in PCM. To overcome this problem, Delta Modulation is used.

Working Principle:

Delta modulation transmits only one bit per sample. Here, the present sample value is compared with the previous sample value and this result whether the amplitude is increased or decreased is transmitted. Input signal $x(t)$ is approximated to step signal by the delta modulator. This step size is kept fixed i.e., $+\Delta$ and $-\Delta$.

The difference between the input signal $x(t)$ and staire case approximated signal is confined to two levels i.e $+\Delta$ and $-\Delta$

Now, if the difference is positive, then approximated signal is increased by one step i.e Δ. If the difference is negative, then approximated signal is reduced by 'Δ'.

When the step size is reduced '0' is transmitted and if the stepsize is increased '1' is transmitted. Hence for each sample, only one bit is transmitted. Fig 15 shows the analog signal $x(t)$ and its staircase approximated signal by the delta Modulator.

<u>Mathematical Expressions.</u>

The error between the sampled value of $x(t)$ and last approximated sample is given as

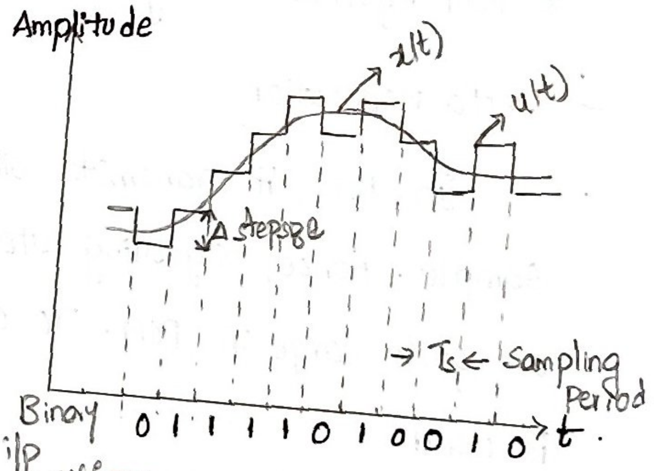


Fig 15: Delta modulation waveform

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

where $e(nT_s)$ = error at present sample

$x(nT_s)$ = sampled signal of $x(t)$

$\hat{x}(nT_s)$ = last sample approximation of the staircase waveform

If we assume $u(nT_s)$ as the present sample approximation of staircase output, then

$$u[(n-1)T_s] = \hat{x}(nT_s)$$

: last sample approximation of staircase waveform.

Let us define a quantity $b(nT_s)$ in such a way that,

$$b(nT_s) = \Delta \, sgn\,[e(nT_s)]$$

$$b(nT_s) = \begin{cases} +\Delta & \text{if } x(nT_s) \geq \hat{x}(nT_s) \\ -\Delta & \text{if } x(nT_s) < \hat{x}(nT_s) \end{cases}$$

Also if $b(nT_s) = +\Delta$ then a binary '1' is transmitted

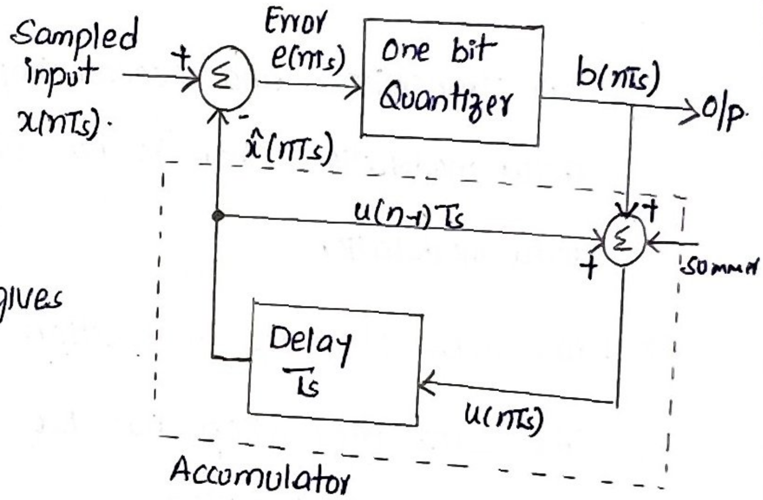
$b(nT_s) = -\Delta$ then a binary '0' is transmitted

Here T_s = sampling interval.

Transmitter part:

Fig16 shows the transmitter.

The Σ i.e summer in the accomulator adds quantizer output ($\pm\Delta$) with the previous sample approximation. This gives present sample approximation



$$u(nT_s) = u(nT_s - T_s) + [\pm\Delta]$$

(or) $$u(nT_s) = u(n-1)T_s + b(nT_s)$$

Fig 16: Delta Modulation transmitter

The sampled input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$

Receiver part:

The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s. It is then added to the input signal.

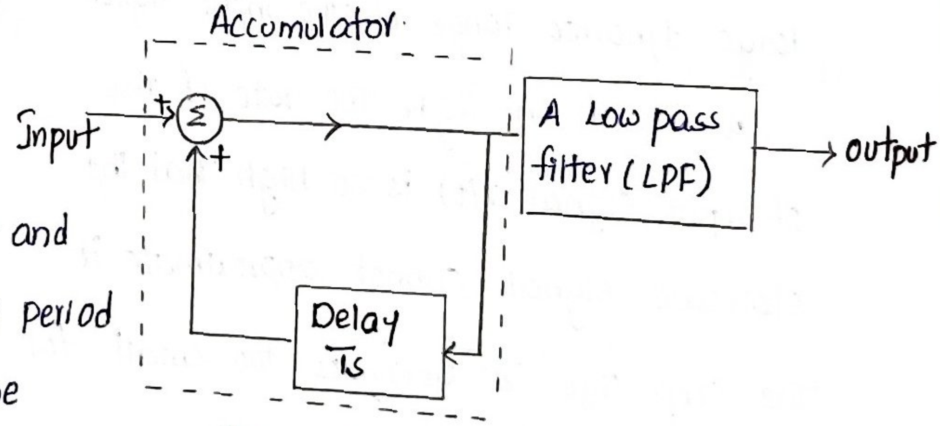


Fig 16(b) Delta modulation receiver

If the input is binary '1' then it adds $+\Delta$ step to the previous output and if the input is binary '0' then one step 'Δ' is subtracted from the delayed signal. Then low-pass filter smoothens to staircase signal to reconstruct original message signal $x(t)$.

→ Advantages of Delta Modulation

1. The delta modulation transmits only one bit for one sample, therefore the signaling rate and transmission channel bandwidth is quite small for delta modulation compared to PCM.

2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter required in delta modulation

→ Drawbacks of Delta modulation

The delta modulation has two major drawbacks as under:

1. slope overload distortion
2. Granular Noise.

→ Slope overload Distortion

This distortion arises because of large dynamic range of the input signal. As shown in fig 17 the rate of rise of input signal x(t) is so high that the staircase signal cannot approximate it.

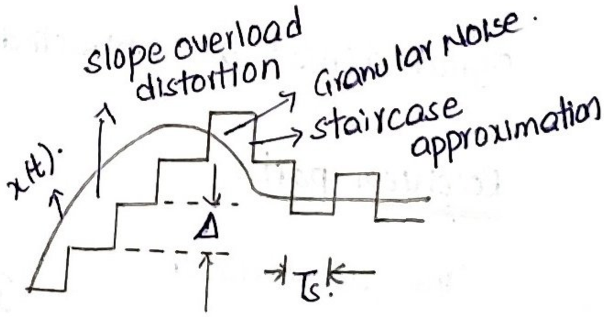


Fig 17 Quantization errors in Delta Modulation

The step size 'Δ' becomes too small for staircase signal u(t) to follow the step segment of x(t).

Hence, there is a large error between the staircase approximated signal and the original signal x(t). This error or noise is known as "slope overload distortion".

(ii) Granular (or) Idle Noise

Granular or Idle noise occurs when the step size is too large compared to small variations in the input signal. This means that for very small variations in the input signal, the staircase signal is changed by large amount (Δ) because of large step size.

Fig shows that when the input signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by $\pm\Delta$ around the signal. The error between the input and approximated signal is called "Granular Noise".

→ **Bit Rate (i.e Signaling Rate) of Delta Modulation**

Delta Modulation bit rate (r) = Number of bits transmitted/second

= No. of samples/sec × No. of bits/sample

= $f_s \times 1$

= f_s

P→ Given a sine wave of frequency f_m and amplitude A_m applied to a delta modulator having step size Δ. Show that the slope overload distortion will occur if

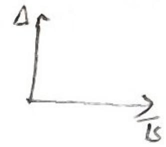
$$A_m > \frac{\Delta}{2\pi f_m T_s}$$ here T_s is the sampling period.

Sol: Let us consider that the sinewave is represented as

$$x(t) = A_m \sin(2\pi f_m t)$$

It may be noted that the slope of $x(t)$ will be maximum when derivative of $x(t)$ with respect to 't' will be maximum.

The maximum slope of delta modulator may be given as



$$\text{Maximum slope} = \frac{\text{step size}}{\text{Sampling period}} = \frac{\Delta}{T_s}$$

$y = mx + c$

$\Delta = mT_s$

$m = \frac{\Delta}{T_s}$

We know that, slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator i.e

$$\max\left|\frac{d}{dt}x(t)\right| > \frac{\Delta}{T_s}$$

$$\max\left|\frac{d}{dt}(A_m \sin 2\pi f_m t)\right| > \frac{\Delta}{T_s}$$

$$\max\left|A_m \cos(2\pi f_m t)\cdot 2\pi f_m\right| > \frac{\Delta}{T_s}$$

$$\therefore A_m 2\pi f_m > \frac{\Delta}{T_s}$$

$$A_m > \frac{\Delta}{2\pi f_m T_s}$$ Hence proved

→ A delta modulator system is designed to operate at five times the Nyquist rate for a signal having a bandwidth equal to 3kHz bandwidth. Calculate the maximum amplitude of a 2kHz input sinusoidal signal for which the delta modulator does not have slope overload. Given that the quantizing step size is 250mV. Also derive the formula that you use.

Sol In the above example, we have derived the relation for slope overload distortion which will occur if

$$A_m > \frac{\Delta}{2\pi f_m T_s}$$

Thus, slope overload distortion will not occur if $A_m \le \frac{\Delta}{2\pi f_m T_s}$

The maximum frequency in the signal is $f_m = 3KHz$

We know the Nyquist rate is $f_s = 2f_m$

$$= 2 \times 3 \times 10^3 = 6KHz$$

Given sampling frequency = 5 times of Nyquist rate

$$f_s = 5 \times 6K = 30KHz$$

Hence, sampling interval

$$T_s = \frac{1}{f_s} = \frac{1}{30 \times 10^3} \text{ sec}$$

Given $\Delta = 250mV$, $f_m = 2 \times 10^3 Hz$

Substituting all the values in $A_m \leq \frac{\Delta}{2\pi f_m T_s}$

$$\leq \frac{250 \times 10^{-3}}{2 \times 3.14 \times 2 \times 10^3 \times \frac{1}{30 \times 10^3}}$$

$$\leq 0.6V.$$

→ **Maximum Signal to Noise ratio in Delta Modulation**

We know the condition to avoid the slope overload distortion is

$$A < \frac{\Delta}{2\pi f_m T_s} \quad \text{—(1)}$$

Therefore, the maximum value of the output signal power is

$$P_{max} = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

$$= \left(\frac{\Delta}{2\pi f_m T_s}\right)^2 \times \frac{1}{2} = \frac{\Delta^2}{4\pi^2 f_m^2 T_s^2} = \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2} \quad \text{—(2)}$$

The quantization error in delta modulation is equally likely to lie anywhere in the interval $(-\Delta, \Delta)$.

This error is assumed uniformly distributed random variable.

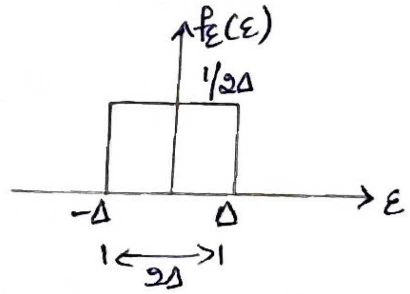


Fig : PDF of Quantization

Thus, the PDF of uniform distribution function can be defined as

$$f_\varepsilon(\varepsilon) = \begin{cases} \frac{1}{2\Delta} & \text{for } -\Delta \le f_\varepsilon(\varepsilon) \le \Delta \\ 0 & \text{otherwise} \end{cases}$$

The mean square value of the quantization noise is given by

$$\overline{\varepsilon^2} = \int_{-\Delta}^{\Delta} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

$$= \int_{-\Delta}^{\Delta} \varepsilon^2 \cdot \frac{1}{2\Delta}\, d\varepsilon = \frac{1}{2\Delta}\left[\frac{\varepsilon^3}{3}\right]_{-\Delta}^{\Delta}$$

$$= \frac{1}{6\Delta}\left[\Delta^3 + \Delta^3\right] = \frac{2\Delta^3}{6\Delta} = \frac{\Delta^2}{3}$$

Therefore, Normalized quantization noise power $N_q = \overline{\varepsilon^2} = \frac{\Delta^2}{3}$ —(3)

The delta modulated signal is passed through a reconstruction low pass filter (LPF) at the output of a DM receiver.

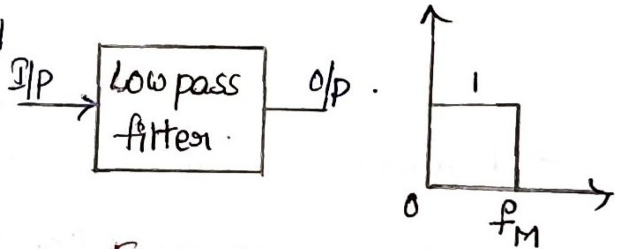


Fig 18: Low pass filter (LPF) at delta

The bandwidth of this LPF is f_M such that

$f_M \ge f_m$ and $f_M << f_s$

Now, assuming that the quantization noise power N_q is distributed uniformly over the frequency band upto f_s, the output quantization noise power with in the bandwidth f_M is given by

Normalized noise power at the filter output,

$$N_q' = \frac{\Delta^2}{3} \times \frac{f_M}{f_s} \quad \text{—(4)}$$

Here, substituting the values from eq (2), (4), we obtain the expression for output signal to quantization noise ratio as under:

$$\left[\frac{S}{N}\right]_0 = \frac{P_{max}}{N_q'}$$

$$= \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2} \times \frac{3f_s}{\Delta^2 f_M} = \frac{3f_s^3}{8\pi^2 f_m^2 f_M}$$

$$\left[\frac{S}{N}\right]_0 = \frac{3}{8\pi f_M f_m^2 \cancel{f_s^3}}$$

P→ A sinusoidal voice signal $x(t) = \cos(6000\pi t)$ is to be transmitted using either PCM or DM. The sampling rate for PCM system is 8KHz and for the transmission with DM, the step size Δ is decided to be of 31.25mV. The slope overload distortion is to be avoided. Assume that the number of quantization levels for a PCM system is 64. Determine the signaling rates of both these systems and also comment on the result.

Sol Given $x(t) = \cos(6000\pi t)$

f_s of PCM $= 8KHz$

$\Delta = 31.25mV$

No. of quantization levels in PCM : $q = 64$:

Signalling rate of PCM & DM ?

We know that the signalling rate in PCM system is

$$r = v f_s$$

$q = 64$

$2^v = 64$

$v = \log_2 64 = \log_2 2^6 = 6$

$r = 6 \times 8 \times 10^3$

$= 48KHz$

The signaling rate of a Delta Modulation is equal to its Sampling rate f_s. We know that the condition to avoid the slope overload distortion is given by

$$A \leq \frac{\Delta}{2\pi f_m T_s} \Rightarrow A \leq \frac{\Delta f_s}{2\pi f_m}.$$

$$f_s \geq \frac{2\pi f_m A}{\Delta}.$$

Given $x(t) = \cos(6000\pi t)$

$A = 1V$, $\quad \omega_m = 6000\pi$

$f_m = 3000 = 3KHz$.

$$f_s \geq \frac{2\pi \times 3 \times 10^3 \times 1}{31.25 \times 10^{-3}}$$

$f_s \geq 603.18 KHz$.

Therefore, signalling rate of DM $\geq 603.18 KHz$.

Comment : To transmit the same voice signal, the DM needs a very large signalling rate as compared to PCM. This is the biggest drawback of DM, which makes it an impractical system.

→ Determine the output signal-to-noise ratio of a linear delta modulation system for a 2KHz sinusoidal input signal sampled at 64KHz. Slope overload distortion is not present and the post reconstruction filter has a bandwidth of 4KHz.

Sol) Given data is $f_m = 2KHz$

$f_s = 64KHz$

$f_M = 4KHz$.

We know that the $(SNR)_o$ of delta modulation is

$$(SNR)_o = \frac{3 f_s^3}{8\pi^2 f_m^2 f_M}$$

$$= \frac{3 \times (64 \times 10^3)^3}{8\pi^2 \times (2 \times 10^3)^2 \times 4 \times 10^3} = 622.51$$

$$(SNR)_o \text{ in dB} = 10 \log_{10} (622.51)$$

$$= 27.94 \text{ dB}.$$

→ For the same sinusoidal input of above problem. Calculate the signal to quantization noise ratio of a PCM system which has the same data of 64kbits/sec. The sampling frequency is 8kHz and the no. of bits per sample is N=8. Comment on the result.

Sol) The signal to Noise ratio of a sinusoidal signal in PCM system is

Given N = 8 = 2 U.

$$(SNR)_q = (1.8 + 6N) \text{ dB}.$$

$$= (1.8 + 6 \times 8)$$

$$= 49.8 \text{ dB}$$

Comment: The SNR of a DM system is 27.94 dB. which is too poor as compared to 49.8 dB of an 8-bit PCM system. Thus, for all the simplicity of DM, it cannot perform as well as an 8-bit PCM.

→ Adaptive Delta Modulation

1. Reason to use Adaptive Delta Modulation

To overcome the quantization errors due to slope overload and granular noise, the step size (Δ) is made adaptive to variations in the input signal x(t). Particularly in the steep segment of the signal x(t), the step size is increased.

Also, if the input is varying slowly, the step size is reduced. Then, this method is known as "Adaptive Delta Modulation (ADM)".

2. Transmitter Part

Fig 19 shows the transmitter of adaptive data Modulator. The logic for step size control is added in the diagram.

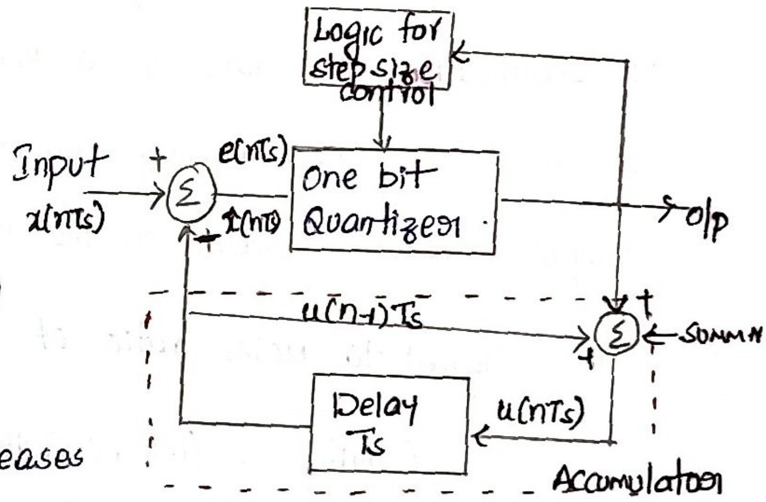


Fig 19: ADM Transmitter

The step size increases or decreases according to a specified rule depending on one bit quantizer output.

As an example, if one bit quantizer output is high (i.e 1), then step size may be doubled for next sample. If one bit quantizer output is low, then step size may be reduced by one step.

Fig shows the staircase waveform of Adaptive delta modulation and the sequence of the bits to be transmitted.

3. Receiver Part:

In the receiver of adaptive delta modulator as shown in fig 20 there are two portions. The first portion produces the step size from each incoming bit. Exactly the same process is followed as that in the transmitter. The previous input and present input decides the step size. It is then applied to the accomulator which builds up staircase waveform. The low pass filter then smoothens out the staircase waveform to reconstruct the original signal.

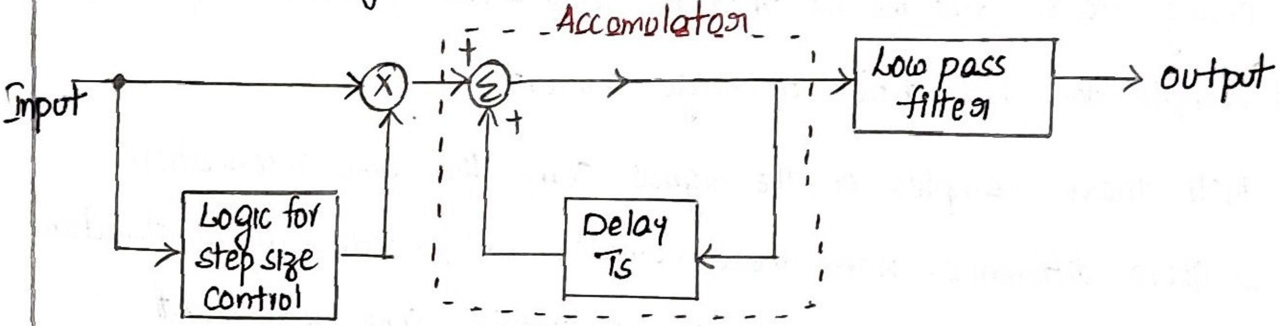


Fig 20 : Adaptive Delta Modulator Receiver.

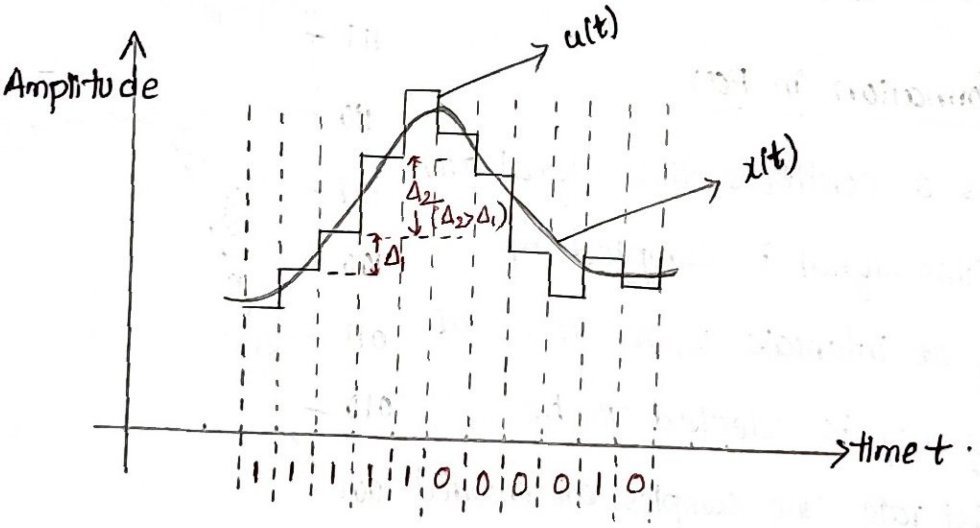


Fig 21: Waveforms of adaptive delta Modulation.

Advantages of Adaptive Delta Modulation

1. The signal to Noise ratio becomes better than ordinary delta modulation because of the reduction in slope over load distortion and idle Noise.
2. Because of the variable step size, the dynamic range of ADM is wider than simple DM.
3. Utilization of Bandwidth is better than delta modulation.

→ Differential Pulse code Modulation (DPCM)

1. Reason to use DPCM

It may be observed that the samples of a signal are highly correlated with each other. This is due to the fact that any signal does not change fast. This means that its value from present sample to next sample does not differ by large amount.

With these samples of the signal carry the same information with a little difference. When these samples are encoded by a standard PCM system, the resultant encoded signal contain some redundant information. Fig22 illustrates this redundant information.

2. Redundant Information in PCM

Fig 22 shows a continous time signal $x(t)$ by dotted line. This signal is sampled by flat top sampling at intervals $T_s, 2T_s, 3T_s \ldots nT_s$. The sampling frequency is selected to be higher than nyquist rate. The samples are encoded by using 3bit (7levels) PCM. The sample is quantized to the nearest digital level as shown by small circles in the figure 22

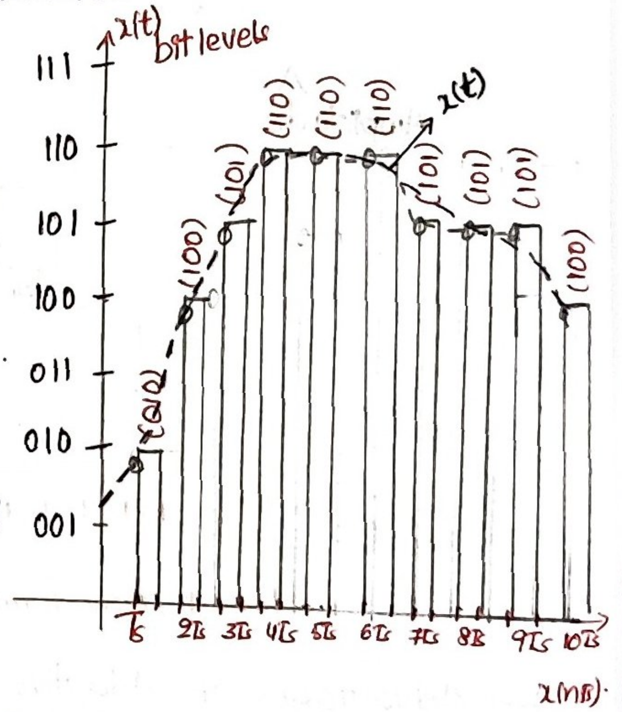


Fig22: Illustration of redundant information in PCM.

The encoded binary value of each sample is written on the top of the samples. We can observe from fig22 that the samples taken at $4T_s$, $5T_s$ and $6T_s$ are encoded to same value of (110).

This information can be carried only one sample. But three samples are carrying the same information means that it is "redundant".

~~The~~ We consider another example of samples taken at $9T_s$ and $10T_s$. The difference between these samples only due to last bit and first two bits are redundant, since they do not change

If this redundancy is reduced, then overall bit rate will decrease and no. of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is known as "Differential Pulse code Modulation (DPCM).

3. **Working Principle**

It works on the principle of prediction. The value of the present sample is predicted from the past samples.

The prediction may not be exact but it is very close to the actual sample value.

Fig23 shows the transmitter of Differential Pulse code Modulation (DPCM) system.

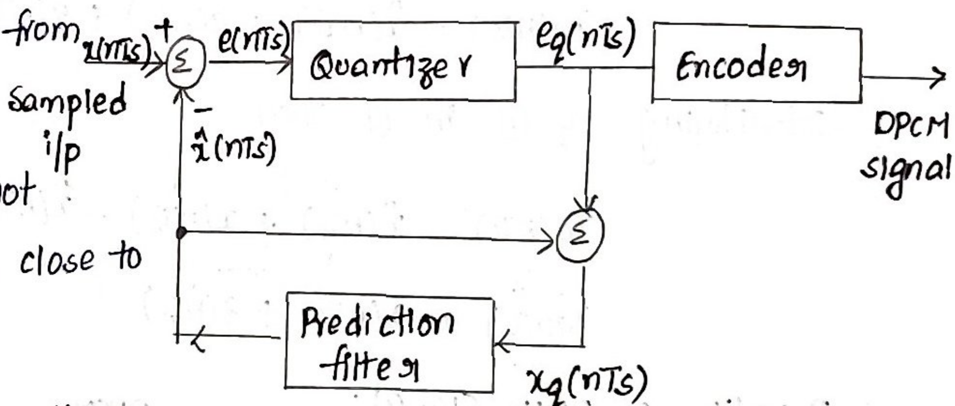


Fig 23: Differential pulse code Modulation

The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$. This is known as Prediction error and it is denoted by $e(nT_s)$. It can be defined as

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad —(1)$$

The quantizer output signal gap $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called as $x_q(nT_s)$

This makes the prediction more and more close to all actual sampled signal. We can observe that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small no. of bits. Thus, no. of bits per sample are reduced in DPCM.

The quantizer output can be written as

$$e_q(nT_s) = e(nT_s) + q(nT_s) \quad -② $$

where $q(nT_s)$ is the quantization error.

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \quad -③$$

Substituting eq ② in ③ then

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \quad -④$$

Substituting eq ① in ④ then

$$x_q(nT_s) = \hat{x}(nT_s) + x(nT_s) - \hat{x}(nT_s) + q(nT_s)$$

$$x_q(nT_s) = x(nT_s) + q(nT_s)$$

4. Reception of DPCM signal:

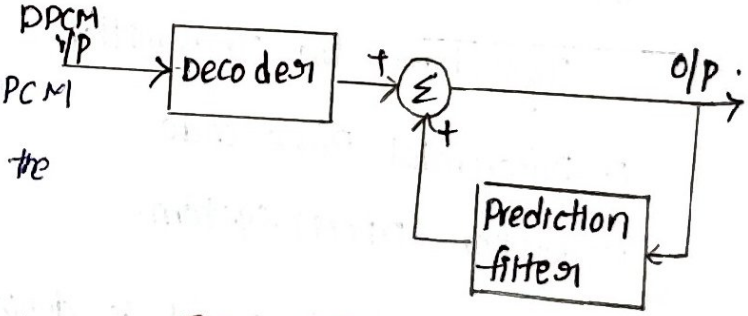


Fig 24: DPCM Receiver.

Fig 24 shows the block diagram of DPCM Receiver. The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter output and quantized error signals are summed up to give the quantized version of the original signal. Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

Comparison between PCM, Delta Modulation, Adaptive Delta Modulation and Differential Pulse code Modulation

S.No.	Parameter of comparison	Pulse code Modulation (PCM)	Delta Modulation (DM)	Adaptive Delta Modulation (ADM)	Differential Pulse code Modulation (DPCM)
1.	No. of bits	It can use 4, 8, or 16 bits per sample	It uses only one bit for one sample	only one bit is used to encode one sample	Bits can be more than one bit but less than PCM
2.	Levels and step size	The no. of levels depend on no. of bits. Level size is kept fixed	step size is kept fixed and cannot be varied	According to the signal variation, step size varies	Here, Fixed no. of levels are used.
3.	Quantization error and distortion	Quantization error depends on no. of levels used	Slope overload distortion and Granular noise are present	Quantization noise is present but other errors are absent	Slope overload distortion and quantization noise is present.
4.	Transmission Bandwidth	Highest BW is required since no. of bits are high	Lowest BW is required	Lowest BW is required	Bandwidth required is lower than PCM.
5.	Feedback	There is no feedback in transmitter or Receiver	Feedback exists in transmitter	Feedback exists	Feedback exists
6.	Complexity of implementation	System Complex	Simple	Simple	Simple.